We want to convey our thanks to the recommender and all reviewers for the helpful comments. Most comments have been implemented precisely as suggested. The reasons for not implementing the remaining comments are listed explicitly below.

Answers to Comments by the Recommender

> line 5: is $n-2$ --> is at most $n-2$

Actually, we wanted to express that, for each n, there are examples needing $n-2$ reticulations. As such, "at least $n-2$" would be more appropriate, but "at most $n-2$" is also true. Thus, we consider "is $n-2$" the correct formulation here.

> line 53: Mention briefly what a universal network is (before saying how it can be constructed).

What we write in line 53 actually **is** the definition of "universal tree based network".

> line 75: What does 'its' refer to?

We changed "If X is a MUL-network, a subset of its nodes, or a sequence" to "If X is a MUL-network, a subset of nodes in a MUL-network, or a sequence".

> line 93: unique minimum --> unique minimum node

We disagree on this point. In our humble opinion, "The minimum of a set S with respect to a partial order" is a fairly rigid mathematical concept and we think that, if S is a set of nodes, it should be self-explanatory that the minimum too, is a node.

> Has your construction to find triples of caterpillars that are 'as different as possible' any connections to work on strings and sequences? You already comment that the caterpillar construction problem is essentially a problem on finding strings that have short common subsequences. Is anything known about how short these subsequences can get for a fixed number of sequences that consist of the same (multi)set of letters?

Thanks for pointing this out. We added a paragraph in the discussion-section about the relation of our work to research on shortest supersequences (corresponding, roughly, to caterpillars with multiple occurances) of permutations (corresponding, roughly, to caterpillars without multiple occurances of leaves). While stronger bounds than ours are known in this case, their application to our problem requires proving that there is always an optimal MUL-caterpillar displaying all input caterpillars, which is left open for future work.

Answers to Comments by Reviewer 1

> For the case $k=2$, which was solved in 2005, the answer is $n-2$, meaning that for two phylogenetic trees, there always exists a network with at most $n-2$ reticulation vertices displaying both trees.

We want to point out that, what's important for our work here, is the **lower bound** of $n-2$ showed by Baroni, Semple, and Steel, not the upper bound! That is, for any n, there are two phylogenetic trees that **cannot** be displayed by a network with **strictly fewer** than $n-2$ reticulations.

> Looking at the induction in the proofs of Lemma 1 and Lemma 2, I think you should state in the Preliminaries that a single isolated network is considered a MUL-network (or a single isolated edge, whichever you prefer). Your current definition implies that a MUL-tree necessarily has two or more leaves, which conflicts with the base case of your induction in both lemmas.

We changed the definition of "MUL-network" to allow the root to have out-degree zero and leaves to have in-degree zero.

> In the proof of Claim 1, case 2: I am wondering about the leaves of N_r that are not in L_Q. Should we not have a case 2a', where none of the other two caterpillars are embedded in a leaf of L_Q, but are both embedded in a leaf of $N_r - L_Q$? If this situation cannot happen, then the reason is not obvious to me.

This situation is covered by Case 2a ("all caterpillars with a leaf embedded into a leaf of L_Q have the same parity as Q") since, in the situation you are describing, the all-quantification is over the empty set, and so the condition for Case 2 is satisfied.

Answers to Comments by Reviewer 2

> l.278: Add 'and' between 'that' and 'leaves'.

We do not think this would be grammatically correct.