

Dear editors and reviewers,

We very much thank the reviewers and the recommender for their careful reading of the manuscript and very valuable comments and suggestions. Their comments were most helpful for us to improve our manuscript. Please find below some comments/suggestions mentioned in the reports, followed by our answers. We have written the main changes in red color in the revised version of the manuscript.

1. REVIEWER #1

We are very happy to read this report. It seems to indicate that we reached the goals we set to ourselves in taking up this work. We are thankful to the reviewer for it. We answered all of the points raised by reviewer 1 in the response below.

- (Comment 1 of report 1) *P.5, The coupling $\langle \nu, \phi \rangle$ is not well-defined in general if ν is a measure and ϕ is only assumed to be bounded measurable. I suggest to restrict ϕ to continuous functions here.*

Since we only work with finite measures $\nu \in M_F(\omega)$ here, assuming that ϕ is bounded and measurable is enough to ensure that $\langle \nu, \phi \rangle$ is well-defined.

- (Comment 2 of report 1) *P.5, Is f_t uniquely determined by (5)? If so, please state it in the Proposition 2.1 with appropriate references. The same comments applies for Propositions 2.3 and 2.4.*

All these equation indeed define a unique measure-valued solution. We added the appropriate references when needed.

- (Comment 3 of report 1) *P.6, 1st line, "if f_0 is absolutely continuous with respect to the Lebesgue measure,...."*

This has been corrected.

- (Comment 4 of report 1) *P.13, the last paragraph. I think the explanation for the initial bias to the right is well written. Can you give more detail to the explanation for the other case when the initial bias is to the left? Does this has to do with the monotonicity of the eigenvalue perhaps?*

We added the following text (p. 14): "Due to mutations, the population cannot remain at this optimum. As the individuals on the left have a smaller mutation rate, they tend to remain closer to the optimum, and therefore increase in proportion compared to the individuals on the right. This explains the initial bias towards left. As we will see with the spectral analysis in the next section, this bias towards left becomes a general feature at large times, independently of the conditions in Proposition 3.1."

- (Comment 5 of report 1) *P.14. It seems to be that there are two different mechanisms at work – the most dominant force is the fitness gradient, which forces the population to move towards x_1 and x_2 (and nowhere else). When the population is away from x_1, x_2 , the two fitness optimal is roughly the same, so the mutation asymmetry takes the population towards x_1 . When the population is concentrated in a neighborhood of $\{x_1, x_2\}$, the model becomes one of competition between two distinct population, in which the population around x_2 wins by virtue of a smaller variance (or the monotonicity of eigenvalue).*

We think that the reviewer is talking about \mathcal{O}_b and \mathcal{O}_s (“ x_1 and x_2 ”). We totally agree with this point of view, which is related to our explanations in page 14 (see our answer to Comment 4).

We have corrected all of the typos that the reviewer highlighted. Thank you for your careful reading of the manuscript.

2. REVIEWER #2

We very much thank the reviewer for their very relevant comments on our work.

(Comment 1 of report 2) *They examined the case where the landscape of the birth rate $b(x)$ and that of the survival rate $s(x)$ have the same functional with different maximum points (equations 9 and 33). It would be great if they can take a glance at the case of strict trade-off between the birth rate and survival rate, where $b(x) + s(x)$ is constant.*

Thank you for the interesting comment. If the fitness function is constant, mutations play an ever stronger role as the trajectories of adaptation will not follow a fitness gradient. But on the other hand, there is no “mutation load” in the strict sense, as the fitness is the same everywhere.

In fact, this configuration corresponds to our “analytically tractable example” in page 17. We did not emphasised this point in the previous version of the manuscript. In this new version, we added several sentences to underline this point (pages 17 and 18). Additionally, we observed that in general, if m is constant, then the equilibrium distribution is proportional to $1/b$. This means that the equilibrium distribution takes larger values when b is small and therefore promotes the high survival strategy. We explain this in page 16, and give some details in the new Appendix A.4, together with new numerical computations (Appendix A.4) to illustrate the dynamics of adaptation in this case. Interestingly, in spite of the absence of mutation load, adaptation still tends to promote the survival strategy. We think that this is the consequence of higher fluxes from the regions of the phenotype space with a high birth rate (and therefore a high mutation rate) to the regions with a high survival rate.

We are very grateful to the reviewers and recommender for their very valuable and useful comments and suggestions. We hope the revised version meets their expectations.

Sincerely yours,

Florian Patout, Raphaël Forien, Matthieu Alfaro, Julien Papaix and Lionel Roques